Michael Friedman works in the Cluster of Excellence "Matters of Activity. Image Space Material" at Humboldt University in Berlin. (Photo credit: Michael Lorber)

EPISTEMOLOGIST AND HISTORIAN OF MATHEMATICS MICHAEL FRIEDMAN, PHD.

What is Fold(ing)?

Interview by Laura Rozenberg

Michael Friedman might be the right person to answer this question. He is an Israeli epistemologist and historian of mathematics living in Germany and has recently published a book, A History of Folding in Mathematics. Mathematizing the Margins ***(Birkhäuser, 2018). Standing at the crossroads of math and the history of science, the book tackles a problem that has been puzzling paperfolders and scientists alike: what exactly is a fold and why it took so long for mathematicians to recognize its value in the sciences? In the following interview, conducted by email, Friedman discussed some of these issues with*** The Paper.

Tell us about your background and how you became interested in the subject of folding, and more specifically, in the history of folding in Mathematics.

After finishing my master in philosophy and my PhD in mathematics, and doing two post-docs in pure mathematics, the first in Bonn, Germany and the second in Grenoble, France, I was invited by Prof. Dr. Wolfgang Schäffner to Berlin, to be a post-doc at the interdisciplinary Institute "Image Knowledge Gestaltung", which was part of the Humboldt University in Berlin. I was there a member in a group called "Science of Structures and 3-D Code". The main question that we dealt with – generally formulated – was to explore how three-dimensional structures shape our thinking from various points of view, and if these structures can be thought in terms of code, be that either digital or analog code[1] (The DNA "code", whose spatial, double helix structure is essential, is a good example for this conception).

One may suggest that folded structures stand in contrast to a simple digital code, which can be thought as one-dimensional (i.e. as associating a sequence of letters to another sequence of letters). One of the objects we discussed at the beginning of our research was the problem of protein folding, where the inquiry regarding the folded three-dimensional structure of proteins and the formation of this structure stood at the center. At that time I also started reading books and articles by Erik Demaine, Joseph O'Rourke and Thomas Hull, among others, and I noticed that while there was a good description of the different approaches concerning the protein folding problem in the history of science, when it came to the history of mathematics in general and the history of mathematical paper folding in particular, there was hardly any comprehensive account that would consider the main players, their motivation, the image of mathematics they had and their possible connections.

Sure, Margherita Beloch Piazzolla was well known in the community of mathematical paper folding, as the one who showed in 1934 that one can construct a segment whose length is the third root of two, a problem known as one of the three Delian problems, which cannot be solved with a straightedge and compass. Also Tandalem Sundara Row was well known for his novel approach to mathematical paper folding in his 1893 book *Geometric Exercises in Paper Folding.* One can also name Albrecht Dürer, who in 1525 pointed explicitly towards a systematic usage of folding within mathematics, used in order to fold polyhedra (see the figures) or to prompt (new) mathematical knowledge. But a systematic study about these mathematicians (and many others) together, was not available. It struck me that not only was there a story to tell within the history of mathematics, but also that one should pose seriously a historical and a historiographical question: why weren't this practice and its history taken into account (or hardly taken into account)?

These two questions – the marginalization of this material practice within mathematics, as well as the marginalization of it within the history of mathematics – fascinated me. Especially what fascinated me is the fact that on the one hand one had for ages, at least theoretically, a very simple material for mathematical practice, which enables the construction of segments easily and beautifully (i.e the Beloch's construction), which a geometry based on straightedge and compass was not able to construct. That is, this material practice was mathematically *epistemic*[2], it was prompting new horizons of knowledge. On the other hand, the mathematical community somehow hardly took this knowledge into account, which resulted that it was not until 1934 Beloch discovered her construction. Exactly this dissonance inspired me to continue researching this theme.

"Mathematics (…), does not only creates or produce new domains of knowledge. (…) It is found in a constant process of transformation, including of its own objects, in which this transformation also entails the marginalization of knowledge. The fold, and how it was conceptualized within mathematics, is an exemplar of such marginalization."

(From: A History of Folding in Mathematics, p. 4, by Michael Friedman).

Albrecht Dürer, an artist and mathematician of the Rennaissance, studied the geometry of falling drapery and was first to present unfolded polyhedra systematically in history. The engraving "Melencolia I" (1514), shows a truncated rombohedron, created during the time when he was studying the "nets", that is, the result of unfolding a geometrical solid placing all of its sides in one plane. (Credit: Wellcome Libray, London. "Melencolia I", after Albrecht Dürer. Https://wellcomeimages.org (Copyrighted work available under Creative Commons Attribution only licence CC BY 4.0).

What is a fold? Is it a math operation like addition or multiplication? Is it a geometrical element, like a tangent or an angle? And is it important to codify it?

This is an excellent question. Assuming we deal with folding a piece of paper, then one has to take into account that folding is a *performative* action, in the sense that it eventually creates a crease, that is, a straight line, which is – after this action is completed – called a fold. So it is an operation, but not like multiplication, where the numbers, which are being multiplied, are already given, and one obtains eventually a new number. Here one only has a piece of paper, and the crease appears as if out of nowhere, and may appear everywhere. With multiplication, there is only one result, and this result is certainly not arbitrary… So there is here a strange oscillation between the operative character of the fold and the ideal character of it (i.e., in the sense that it creates ideal objects – the line), between the operation and the obtained object itself. This oscillation emerges also when one looks at folds of drapery, which do not create creases. To give one example, Dürer, in the 16th century, tried a few times to sketch a geometrical construction, something that might even resemble scaffolding, outlining the geometry which underlies the folds of a fabric, but it is clear that his attempts did not lead to a coherent, complete theory. Maybe one of the difficulties with the folds of a fabric, a difficulty which might be more philosophical, is to determine when a fold begins and when it ends. So a question arises: from where should start to mathematize the falling (drapery) of the fold? I think it may also point to the difficulties of "coding" the folding, i.e. finding a "standard" notation system. Returning to fold patterns, consisting of simple, straight creases, only during the 20th century (!) the standard notation system was accepted, using the Yoshizawa system supplemented by Randlett-Harbin. One can also say that to codify a crease pattern means to find computer programs that when one sketches a three-dimensional figure, then the computer program returns, as an output, the two-dimensional crease pattern and the instructions of how one should fold in order to obtain from this crease pattern the three-dimensional object. This is actually the goal of Robert J. Lang's TreeMaker computer program.

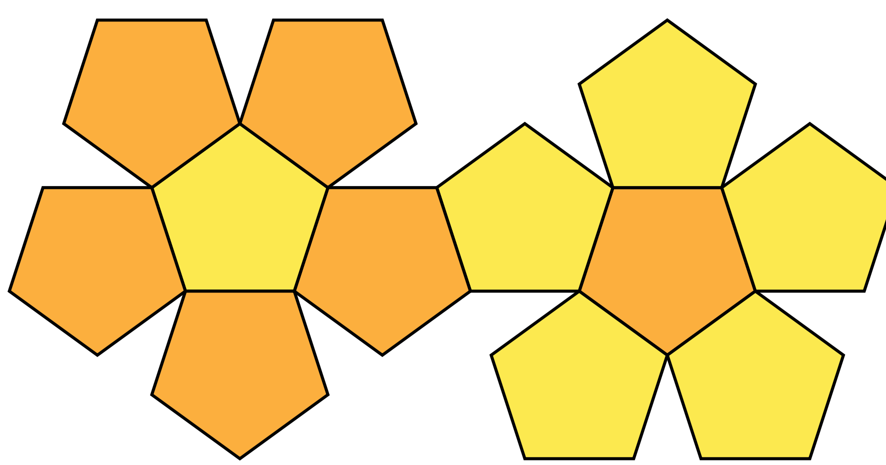
You seem to have found a new domain of thought, almost virgin. For someone immersed in the philosophy of science, finding a field like this one is like stumbling upon a giant diamond in the middle of the desert. No one seems to have seen it, or if they did, they turned away. How did you feel when you started studying the philosophy of folding?

One should recall that the philosophy of folding was already treated by Gilles Deleuze in 1988, in his book *The Fold: Leibniz and the Baroque,* which was published in French (as *Le Pli: Leibnitz et le Baroque*) and was later translated to English. So in a sense a possible epistemological framework was already offered. One can suggest therefore that the philosophical community was already aware of this image of thought, i.e. of thinking explicitly on matter and thought as inherently folded and unfolded, and as Deleuze's revolutionary reading in Leibniz's writings shows, this image of thought was certainly with us, though sometimes implicitly, for centuries. However, what was lacking was a more historical study, which indeed, as you rightfully indicated, was almost virgin. Whether Deleuze's thinking does correspond to the historical analysis – well, this is a good question, but his thought certainly provides insights; for example, according to Deleuze, the fold as a philosophical image may be seen as what points to a process of constant metamorphosis. This was actually what characterized how I felt: challenged, while attempting to see what is the epistemological framework that is enabled (or disabled) when considering folding as a mathematical practice. The main challenge was also practical: to find mathematicians and scholars who indeed employed this material practice of folding in either an epistemic way or in a systematic way[3]. As I mentioned, several mathematicians from the beginning of the 20th century were already known (Row, Beloch), but I had to ask, for example, whether the research really began with Row's book, or was he influenced from other traditions, which might have not been necessarily mathematical.

It was indeed a challenge, since the task at the beginning seemed also as if I was trying to find concrete reasons to why a certain way of reasoning had not been chosen in mathematics. Luckily, I did manage to find quite a few mathematicians who valued and used folding within mathematics! But their methods had been marginalized, sometimes even regarded as not being mathematical enough. So in some sense, I felt somehow sorry for these mathematicians, because I was actually telling a story of non-acceptance or rejection of a technique that eventually proved to be quite powerful. Nevertheless this study was extremely interesting, since it showed that one should not ignore – generally speaking – the materiality of mathematical practices (that is, paper, in the case of folding)[4], which might look irrelevant: that is, materiality itself plays a bigger role than one may think, even in the common conception of mathematics as abstract and "immaterial".

You pose a question that has intrigued many for long time: Why was (and still is) folding so rarely taken seriously? Of course, the answer to this question is contained in your book of over 400 pages. But to those who haven't had the opportunity to read it, can you give a hint on what you found?

There are several reasons, few of them are given by the mathematicians as if by passing: paper is "too material"; folding shapes is a children's play or belongs at best to recreational mathematics, and not something mathematicians should consider; or that the mathematical constructions done by paper folding are too complicated for children; or that one cannot prove anything with folding, but only construct; and of course, that paper is after all ephemeral. All these reasons



At the beginning of the sixteenth century, Albrecht Dürer introduced the fold into mathematics as a legitimate mathematical operation for representing Platonic and Archimedean solids unfolded as "nets". This image illustrates a net of a regular dodecahedron. (Image representation by Júlio Reis - Dodecahedron flat.png, made by Cyp from makepolys.c by himself, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=1272003)

Gottfried Leibniz (1646-1716) and Gilles Deleuze (1925-1995) Deleuze laid the foundations for the philosophy of the fold, based on a particular reading of Leibniz's writings. (Photo credits: Leibniz: Christoph Bernhard Francke [Public domain]; Gilles Deleuze: https://www.flickr.com/photos/speedypete/274083307)

"The study of the mathematization of the fold is therefore a study of its marginalization, a study of how the fold –until the end of the twentieth century– did not become a mathematical object."

(From: A History of Folding in Mathematics, p. 6, by Michael Friedman).

were mentioned in articles, books or even historical accounts in one way or another, and certainly operated as hindrance for folding to be taken seriously.

But I believe that there are also two other reasons, more epistemological, of why this practice was hardly considered mathematical: these concern to materiality and conception of space. As I mentioned above, folding paper is eventually a material practice. And as such, it clearly stands in contrast to the common image of mathematics as abstract, the results of which are not dependent of the material or the instrument. Moreover, one hardly needs any instrument at all while folding: whereas for drawing a line or a circle one needs a straightedge and a compass, for folding one only needs the paper itself, and that's it. So one may observe here the emergence of a non-instrument (the paper), which nevertheless gives rise to a straight line (the crease) – one of the "ideal" objects in plane geometry.

The other reason concerns conceptions of space. Obviously, to fold a paper, which is (ideally) two-dimensional, one has to move through three-dimensional space. However, the end result is again an array of one-dimensional lines (the creases) on the two-dimensional plane (the paper); the embedding three-dimensional space and the movement in it are forgotten... One notes here already several conceptions of space, which are quite modern: embedding space, deformation of objects, or movement as legitimate action in it. These implicit conceptions and operations were not always accepted as legitimate in mathematics, so this also might explain why folding was not even mentioned in antiquity as a possible (theoretical) mathematical practice.

Besides you, who else is studying the epistemology of folding and how do you see the future of this field of research?

I already mentioned Deleuze, so his followers certainly do engage with the epistemology of folding. I would like to mention also two collections of papers I co-edited: the first, co-edited with Prof. Dr. Wolfgang Schäffner, *On Folding* (2016, Bielefeld: transcript). The collection shows, from an interdisciplinary point of view, the great potential of taking the fold and folding as an image of thought. The second, co-edited with Dr. Angelika Seppi, called *Martin Heidegger: Die Falte der Sprache* (Martin Heidegger: the fold of the language) (2017, Vienna: Turia + Kant), which deals with the image of the fold in the thought of the philosopher Martin Heidegger.

I believe the future of this field of research lies in considering also other material practices employed in mathematics. As a historian of mathematics, I think folding shows us how these practices should not only be taken seriously as part of the mathematical research, but also how mathematics is done in "real life". But obviously, folding paper is not the only material practice one had and has: knotting, braiding, construction of material (or virtual) three-dimensional models of surfaces are all material practices, which during centuries were either mathematized or helped to understand and visualize better mathematical concepts and objects, and may be thought as opening new mathematical horizons. Returning to folding, I believe also that one should inquire how the latest developments of folding-based mathematics during the 21st century, with computational origami for example, influence our conception of materiality, and how does it reshape our thought regarding this practice.

Footnotes

1. In contrast to digital codification (as a discrete representation or symbolization of information, e.g. the Morse code or the Braille system), analog codification is a continuous representation of codified information enabled by a continuous set of values (for example, voltage signals).

2. I follow here Hans-Jörg Rheinberger's differentiation between epistemic and technical objects. According to Rheinberger, epistemic objects, in contrast to technical objects, embody what one does not yet know, and prompt the emergence of new knowledge (in contrast to technical objects, which function as if everything is already known about them).

3. By "systematic" I mean folding that was used as an explicit method, which was not used just one time or in a singular fashion, but rather explicitly and as an established method, presented as an acceptable part of the mathematical practice that employed it.

4. By materiality of mathematical practices I mean not only paper, but also three-dimensional models, as well as drawings, sketches, and script itself: all of these are examples of how material aspects emerge within mathematics.